Second Law Analysis on Fractal-Like Fin under Crossflow

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A fin is used for enhancing heat-transfer rate from an object to the surrounding fluid with poor heat-transfer coefficients. The analyses for fins with various shapes were discussed extensively and were summarized in several books such as Kern and Kraus (1972).

The fractal concept developed by Mandelbrot (1982) provides a method for describing many objects made of parts similar to the whole in some way (Feder, 1985). Plawsky (1993a, b) recently proposed an analysis for a fractal-like fin under the natural convection condition. The fractal-like fin is defined as a fin with subfins repeatedly extending in a fixed way. Plawsky had shown that within a first few generations of subfins, although the fin efficiency decreased an extent, fin effectiveness improved greatly. This result suggests a simple method for enhancing fin performance, warranting further investigations.

Recently, the second law analysis has influenced the design methodology of various heat- and mass-transfer systems (Bejan, 1982). The major concern for the second law analysis is the entropy generation rate (or system irreversibility). It has been proposed that a minimization of total process entropy generation rate is equivalent to maximize the system effectiveness. Therefore, besides the conventional process design considerations, an optimal process design should also generate entropy at a minimum rate.

When compared with the natural convection condition, the application of an external flow field can usually largely enhance the heat-transfer rate from an object. This article examines the performance and the entropy generation rate of a fractal-like fin under a crossflow.

Analysis

The fin system is basically the same as the contracting fin system proposed in Plawsky (1993a); however, a slightly different index system is employed and some explanations are required. The base fin (length L_o and diameter D_o) comprises the first two subfins indexed $i = 1$ and 2. Note that $D_1 = D_2 = D_o$, and $L_1 = \alpha L_o$. The first generation of branching ($N = 1$) is composed of two subfins; each comprises two subfins indexed $i = 3$ and 4. The scaled rule is $D_3 = D_4 = \beta D_1$, and $L_3 = \gamma L_1$, $L_4 = \gamma L_2$. All further branchings are constructed in a similar manner, forming a two-dimensional fractal-like fin network. Clearly, index i ranges from 1 to n, where $n = 2(N+1)$. All subfins are attached normally to their mother fin. The surrounding fluid is flowing upwards across the fin at a velocity of U_∞ and a temperature of T_∞.

Three geometric parameters, α, β, and γ, and the branching number N account for the fin branching characteristics and lack in conventional fin systems. α is less than unity. The β and γ are also less than unity for a contracting fin.

The fin temperature distribution, fin effectiveness, and efficiency can be found in Plawsky (1993a) and are not stated here for brevity. The base heat flux can hence be obtained via Fourier's law. A dimensionless group characterizing the base heat flow is added as follows:

$$B = \frac{\rho \nu^3 k T_\infty}{q_{b1}^2}. \tag{1}$$

The entropy generation rate for a horizontal cylinder facing a crossflow had been derived as (Bejan, 1982):

$$\dot{S}_{gen} = \frac{q_{bl}(T_{bl} - T_\infty)}{T_\infty^2} + \frac{F_D U_\infty}{T_\infty}, \tag{2}$$

when the temperature difference is small. For a fractal-like fin, the total entropy generation rate is the sum of the entropy generation rates for all subfins, that is,

$$\dot{S}_{gen,tot} = \sum_{\text{all subfins}} \dot{S}_{gen,i}. \tag{3}$$

A dimensionless entropy generation number based on unit amount of heat transport is defined by Bejan (1982) as follows:

$$N_S = \frac{\dot{S}_{gen,tot}}{(q_{bl}^2 U_\infty)/(k \nu T_\infty^2)}. \tag{4}$$

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Assume the entropy generation rate for a subfin indexed i can be estimated by Eq. 2. Substituting Eqs. 2 and 3 into Eq. 4, the result reads:

$$N_s = N_{sH} + N_{sF} = \frac{-k\nu}{\delta_1 C_1 \Omega_1 U_\infty} + \frac{F_{D,\text{tot}} k \nu T_\infty}{q_{bl}^2}, \tag{5}$$

where C_1, δ_1 and Ω_1 are defined in Plawsky (1993a):

$$F_{D,\text{tot}} = \sum_{\text{all subfins}} \frac{1}{2} \rho C_{D,i} D_i L_i U_\infty^2. \tag{6}$$

The drag coefficient and Nusselt number for subfin indexed i are estimated as follows:

$$C_{D,i} \doteq 5.484 Re_i^{-0.246}, \tag{7a}$$

$$Nu_{,i} = \frac{h_i D_i}{\lambda}$$

$$= 0.30 + \frac{0.62 Re_i^{1/2} Pr^{1/3}}{[1+(0.40Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_i}{282{,}000}\right)^{5/8}\right]^{4/5}, \tag{7b}$$

where

$$Re_i = \frac{\rho D_i U_\infty}{\mu}, \tag{7c}$$

which are valid for a horizontal cylinder under a crossflow with Reynolds number ranging from 40 to 1,000.

An optimal fin design/operational condition based on the second law analysis is to search for a parameter set which minimizes Eq. 5. However, since the evaluation for C_1 and $F_{D,\text{tot}}$ both require the information for all subfins, a direct differentiation of Eq. 5 is proven inaccessible. Numerical evaluations are adopted. The parameter sets for the sample calculations demonstrated in the following figures are listed in Table 1.

In a contracting fin system, in order to prevent subfins to overlay each other, the following geometric constraint should hold: if $\alpha \le (1-\beta W)/(\gamma^2-1)$, then $1/2 + \beta(\gamma^3-\beta^3) + \gamma W(\alpha-\gamma^2) \ge 0$ and if $\alpha > (1-\beta W)/(\gamma^2-1)$, then $\beta(\gamma^3-\beta^3) + \gamma W(\alpha-\gamma^2) \ge 0$ with $W = L_0/D_0$.

Results and Discussion

Fin performance

To illustrate the effects of fin branching features and flow condition, the ratio W is fixed as 5.0 if not otherwise mentioned. Reynolds number based on base fin diameter D_0 is used throughout this work.

Figure 1 demonstrates the fin effectiveness and fin efficiency for a contracting fin under natural convection and a forced convection condition of $Re_o = 160$. Three points are noticeable. First, both the fin effectiveness and efficiency decrease with increasing Reynolds number. Secondly, in most cases, the fin effectiveness increases with decreasing α. Thirdly, the enhancement becomes saturated within the first few generations of branching, especially under a forced convection condition.

Table 1. Parameter Sets for Sample Calculations

No.	α	β	γ
1	0.8	0.6	0.6
2	0.4	0.6	0.6
3	0.2	0.6	0.6
4	0.8	0.3	0.6
5	0.8	0.15	0.6
6	0.8	0.6	0.3
7	0.8	0.6	0.15

The first point merely indicates that though the heat transfer is enhanced under flow condition, the benefit is not as large as in natural convection case. The second point is a result for a larger absolute value of $\delta_1 \Omega_1 C_1$ obtained when α is decreased. The third point simply suggests that an infinite addition of subfins gains no infinite benefit. A fin with $N=1$ or 2 is good enough for heat-transfer enhancement purpose.

Fin effectiveness is also found to improve by increasing β and/or γ values, due to a larger heat-transfer area available. From performance analysis, therefore, a fin with one or two generations of branching under a crossflow with as high as possible fluid velocity is both heat-transfer and fabrication favorable. The branching subfins should be of a low α value, and without violation of geometric constraints of large β and γ values.

Entropy generation

The increase of external fluid flow velocity, nevertheless, also increases the drag force. An increase of Reynolds number is therefore not always beneficial as that demonstrated in fin performance analysis.

Figure 2 shows a entropy generation number against Reynolds number plot with branching number as a parameter. For a fixed fin structure, an optimum Reynolds number exists departed from which a higher overall entropy genera-

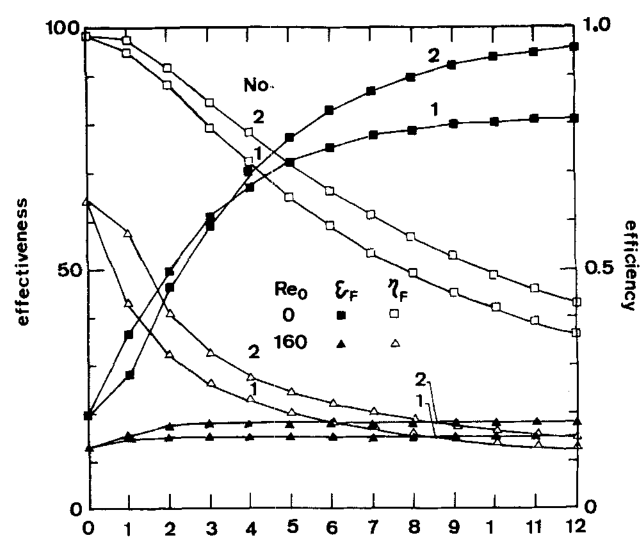

Figure 1. Fin effectiveness and efficiency vs. branching number.

Parameter set No. 1 ($B = 10^{-7}$).

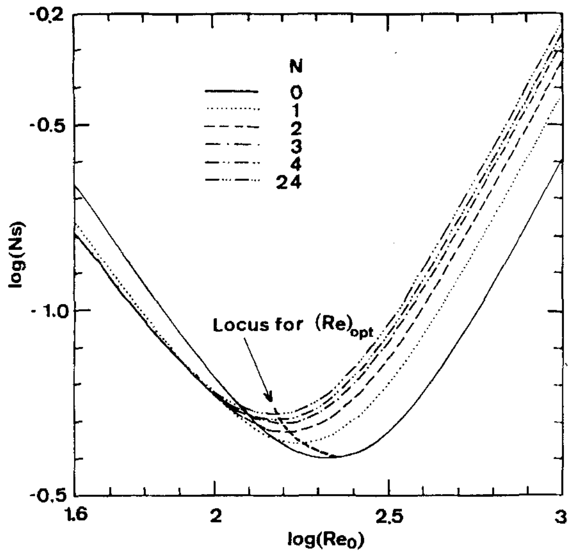

Figure 2. Entropy generation number vs. base fin Reynolds number.
Parameter set No. 1 ($B = 10^{-7}$).

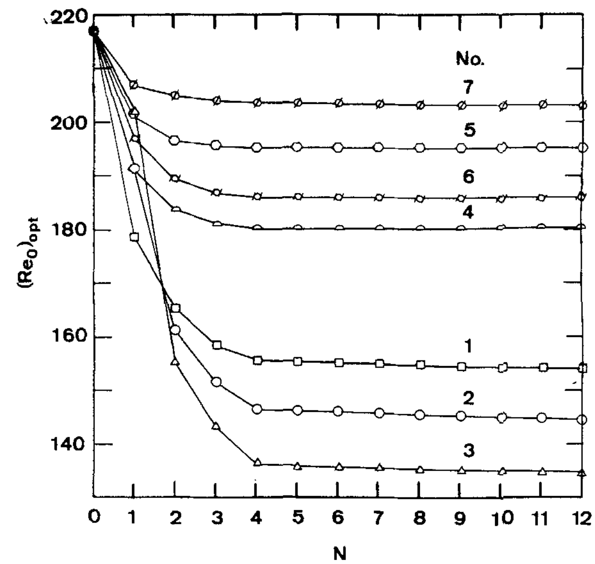

Figure 4. Optimal Reynolds number against branching number.
Parameter set No. 1 to 7 ($B = 10^{-7}$).

tion rate occurs. The position for minimum entropy generation rate can be easily located and the optimal Reynolds number can be found. It is noted that the optimal Reynolds number is higher for a single pin fin ($N = 0$), and decreases and approaches to a plateau value when $N \to \infty$.

Figure 3 demonstrated the two parts of Eq. 5. N_{sH} decreases with increasing Reynolds number, due to the fact that the entropy generation number is based on unit base heat flow and a high heat-transfer rate is associated with a large fluid velocity. Oppositely, N_{sF} increases with fluid Reynolds number, due to a larger fluid drag force. The compromise of the two opposing factors generates an optimal Reynolds number.

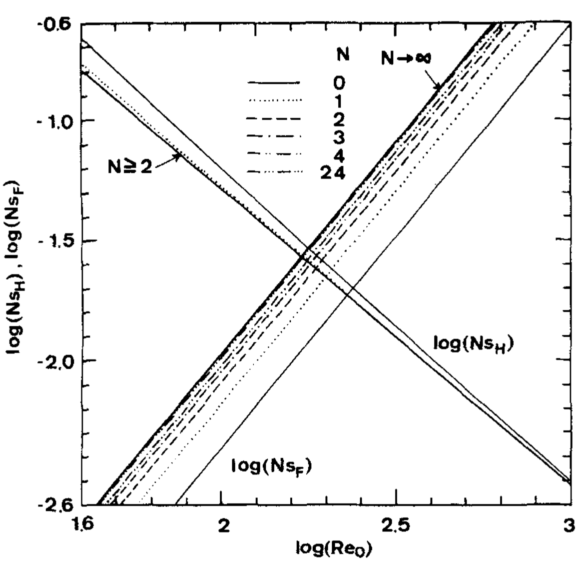

Figure 3. log (N_{sH}) and log (N_{sF}) vs. base fin Reynolds number.
Parameter set No. 1 ($B = 10^{-7}$).

In Figure 3, the effect of increasing branching number on the increase of N_{sF} is more plausible than on the decrease of N_{sH}, which causes the initial decrease of optimal Reynolds number when N is raised. Since both N_{sH} and N_{sF} approach to a constant when $N \to \infty$, an asymptotic optimal Reynolds number is resulted.

Figure 4 demonstrated the optimal Reynolds number for parameter sets No. 1 to 7, with the branching number as the abscissa. As observed in Figure 3, $(Re)_{opt}$ first decreases with increasing branching number, then it approaches to a plateau value when the branching number becomes infinite. The range where optimal Reynolds number varies over a wide range of fin geometric factors is limited.

Since a reduction of parameter α will cause the absolute value of the product of $C_1 \delta_1 \Omega_1$ to increase, a smaller entropy generation number is obtained when α is reduced. It is recommended by the second law analysis that the subfins should be located close to the mother fin base if the approaching crossflow velocity is not affected by such an adjustment, which is also a favorable design proposed by performance analysis. The effect of increase of β and γ on entropy generation number is much more complicated than α does. However, a smaller β or γ are suggested.

Conclusions

The fin performance and entropy generation rate for a fractal-like fin under a crossflow were investigated theoretically. In fin performance analysis, a fin with one or two generations of branching under a crossflow with a high Reynolds number is recommended. The subfins should be of large dimensions, and are located close to their mother fin base. Based on the second law analysis, an optimal Reynolds number exists departed from which a higher overall entropy generation rate occurs. The subfins are also suggested to be located close to the mother fin base; however, small subfin dimensions are recommended.

Acknowledgment

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Notation

B = dimensionless group defined in Eq. 1
C_i = parameters shown in Eq. 5
$C_{D,i}$ = drag coefficient for subfin indexed i
D_i = diameter for subfin indexed i, m
D_0 = diameter for base fin, m
$F_{,D}$ = drag force, N
h_i = heat-transfer coefficient for subfin indexed i, W/m^2·K.
k = thermal conductivity, W/m·K
L_i = length for subfin indexed i, m
L_o = length for base fin, m
n = fin index number
N = branching number
N_{ui} = Nusselt number for subfin indexed i
N_s = entropy generation number defined in Eq. 5
N_{sH} = entropy generation number defined in Eq. 5
N_{sF} = entropy generation number defined in Eq. 5
Pr = Prandtl number
q_{bl} = base heat-transfer rate, W
Re_i = Reynolds number for subfin indexed i
Re_o = Reynolds number for base fin
$\dot{S}_{gen}$ = entropy generation rate, W/K
$T_{,bl}$ = base temperature, K
T_∞ = free-stream temperature, K
U_∞ = free-stream velocity, m/s
W = geometric aspect ratio, $= L_0/D_1$

Greek letters

α = tip section scaling parameter
β = branch diameter scaling parameter
γ = branch section scaling parameter
δ_1 = parameter shown in Eq. 5
λ = fluid thermal conductivity, W/m·K
μ = fluid viscosity, Pas
ν = fluid kinematic viscosity, m^2/s
ρ = fluid density, kg/m^3
Ω_i = dimensionless group shown in Eq. 5

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